

# Lineshape Asymmetry for joint CPT and three photon $N$ resonances

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We show that a characteristic two photon lineshape asymmetry arises in coherent population trapping ( $CPT$ ) and three photon ( $N$ ) resonances because both resonances are simultaneously induced by modulation sidebands in the interrogating laser light. The  $N$  resonance is a three-photon resonance in which a two-photon Raman excitation is combined with a resonant optical pumping field. This joint  $CPT$  and  $N$  resonance can be the dominant source of lineshape distortion, with direct relevance for the operation of miniaturized atomic frequency standards. We present the results of both an experimental study and theoretical treatment of the asymmetry of the joint  $CPT$  and  $N$  resonance under conditions typical to the operation of an  $N$  resonance clock. © 2008 Optical Society of America

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In compact, all-optical atomic clocks employing Coherent Population Trapping ( $CPT$ ) [1–4], or three photon  $N$  resonances [6, 10] interrogating light fields are typically generated by current modulating a single-mode diode laser with a large modulation index. This creates additional optical fields, which can also drive atomic resonances. As we show here both  $N$  resonances [6] and  $CPT$  resonances [6, 7] are typically present. These two processes may then interfere and undergo differential AC-Stark shifts, which lead generally to an asymmetric joint resonance. Temporal variation in lineshape asymmetry has been shown experimentally to lead to clock frequency instabilities [8].

An  $N$  resonance is a three-photon, two-

optical-field absorptive resonance. A probe field  $\Omega_1$ , resonant with the transition between the higher-energy hyperfine level of the ground electronic state and an electronically excited state, optically pumps the atoms into the lower hyperfine level. This probe field,  $\Omega_1$ , also acts on the lower hyperfine state in combination with a drive field  $\Omega_0$  detuned from the probe field by the atomic hyperfine frequency  $h$ . Together,  $\Omega_1$  and  $\Omega_0$  create a two-photon Raman resonance that coherently drives atoms from the lower to the upper hyperfine level. This causes increased absorption of the probe field  $\Omega_1$  in a narrow resonance with linewidth  $\Delta\nu$ , set by the ground-state hyperfine decoherence rate.

A practical  $N$ -resonance clock creates the

fields  $\Omega_0$  and  $\Omega_1$  using modulation of a single laser source. The laser carrier field  $\Omega_0$  is detuned by approximately the ground state hyperfine frequency below the  $F = 2 \rightarrow F' = 1$  transition, and the modulation frequency is set so that the first sideband  $\Omega_1$  is resonant with that transition, leading to an  $N$ -resonance. Additional sidebands such as  $\Omega_2$  (see Fig. 1a) are also present. These sidebands participate with  $\Omega_0$  and/or  $\Omega_1$  in  $CPT$  resonances, which simultaneously compete with the  $N$  resonances, leading to an overall lineshape asymmetry. While an ideal  $CPT$  resonance produced by two optical fields will not have an associated  $N$  resonance, additional optical fields are typically present in  $CPT$  clocks driven by a modulated laser. At the large modulation index used in  $CPT$  clocks [9], higher order sidebands which drive pairs of  $N$  resonances are present. For pure phase modulation of the optical field the asymmetric signals from these  $N$  resonances cancel. In the presence of amplitude modulation this cancellation is incomplete and residual asymmetry may become significant.

Our experimental studies of joint  $N+CPT$  resonances [9–11] used a beam of 795 nm light from a diode laser, tuned near the  $^{87}\text{Rb}$   $D_1$  transition and modulated at the hyperfine frequency by an electro-optic modulator (EOM). The laser light was circularly polarized and sent through a heated  $^{87}\text{Rb}$  vapor cell (65°C, Neon buffer gas of 30 Torr). The vapor cell was housed in high permeability magnetic shields inside of which was a uniform longitudinal magnetic field (used to split the Zeeman degeneracy) created by a solenoid. In results presented here, the  $\Delta m = 0$ , magnetic field-independent transition was studied. A temperature stabilized Fabry-Perot cavity (FP) after the cell selected the +1 sideband, whose intensity was measured with a photodiode. In spectroscopy studies, the synthesizer driving the EOM was locked to a hydrogen maser to provide high frequency stability. Additionally, for elimination of the +2 sideband *before* the cell, the laser beam was retroreflected off a second FP tuned to pass only the +2 sideband. Two-photon lineshapes were measured for various laser detunings and powers, along with

the presence or absence of the +2 sideband. In all data presented, the RF (EOM) modulation index was fixed at 0.6. For example, Figure 1b shows a typical measured lineshape for the transmitted probe field  $\Omega_1$ , illustrating the asymmetry of the joint  $N+CPT$  resonance.

To the usual three-state  $\Lambda$ -system used to model  $CPT$ , we append an additional excited state with dipole coupling to the two ground states. This fourth state is assumed to be far off resonance and accounts for the non-resonant dipole polarizability to which many excited states may contribute. The optical fields  $\Omega_1$  and  $\Omega_2$  form the  $CPT$  system; whereas  $\Omega_0$  and  $\Omega_1$  participate in the  $N$  resonance (see Fig. 1a).  $\Delta$  is the (one photon) laser detuning of  $\Omega_1$  and  $\delta$  is the two-photon detuning of the laser fields for both the  $CPT$  and  $N$  resonances.

Using our model we find that to leading order in the optical fields, the transmission of the probe field  $\Omega_1$  is proportional to  $T(\Delta) - \text{Im}(\rho_{ab} \frac{\Omega_0}{\Omega_1(h-\Delta)})$ . Here  $T(\Delta)$  is the transmission independent of the  $CPT$  and  $N$  resonances,  $h$  is the hyperfine frequency, and  $\rho_{ab}$  is the ground state coherence. In steady state we find,

$$\begin{aligned} \left( \Gamma - i\left(\delta + \frac{|\Omega_1|^2 - |\Omega_0|^2}{4(h-\Delta)}\right) \right) \rho_{ab} = & \\ -i\frac{\Omega_1}{2}\rho_{cb} + i\frac{\Omega_2}{2}\rho_{ac} & \\ + \frac{\Omega_1\Omega_0}{4(h-\Delta)}(\rho_{bb} - \rho_{aa}), & \quad (1) \end{aligned}$$

where  $\Gamma$  is the ground state depolarization rate and the subscripts refer to the atomic levels as shown in Figure 1a. The second line in Eq. (1) is associated with the  $CPT$  resonance and the last line is the contribution of the  $N$  resonance. This expression is derived by adiabatically eliminating the non-resonant states contributing to the atomic polarizability. The equation for the ground state population difference,  $\rho_{bb} - \rho_{aa}$ , is structurally equivalent to Eq. (1), with an  $N$  resonance term proportional to  $\rho_{ab}$  and a  $CPT$  term linear in  $\rho_{ac}$  and  $\rho_{bc}$ . Since atomic coherences (*e.g.*,  $\rho_{cb}$ ) scale as  $\Omega/\Delta$ , the leading order  $CPT$  and  $N$  resonance driving terms are of the same order at relevant one-photon detunings. Note that for the numerically calculated results

presented below, the contribution from the full excited state manifold is utilized, whereas Eq. (1) is for only a single resonant excited state.

Numerical calculations of the probe field  $\Omega_1$  transmission intensity are shown graphically in Figure 2 in the approximation that the vapor cell is optically thin. The limiting cases of pure *CPT* resonance ( $\Omega_0 = 0$ ) and ideal *N* resonance ( $\Omega_2 = 0$ ) are not centered at the same two-photon detuning because they experience different AC Stark shifts. The relative amplitudes for *CPT* and *N* resonances are typical of experimental results.

For comparison between theory and experiment we quantify the lineshape asymmetry by fitting to a skew Lorentzian [12]:

$$I(\delta) = C + D\delta + \frac{A\Gamma + B(\delta - \delta_0)}{(\delta - \delta_0)^2 + \Gamma^2}. \quad (2)$$

Here  $I(\delta)$  is the transmitted  $\Omega_1$  intensity of the combined *N+CPT* resonance,  $\delta$  is the two-photon frequency, and  $A$ ,  $B$ ,  $C$ , and  $D$  are fitting amplitudes. The fitting parameter  $\Gamma$  is proportional to the resonance linewidth, and  $\delta_0$  is the two-photon resonance frequency. The amplitudes  $A$  and  $B$  describe the symmetric (Lorentzian) and antisymmetric (dispersive) components of the lineshape, respectively. We define the line asymmetry as the dimensionless ratio  $B/A$ . As an example, consider the effect of non-resonant light fields: these fields are time-varying in the rotating frame and thus contribute AC Stark shifts to each transition. The sign of these shifts depends on the sign of the non-resonant field's detuning. Different AC Stark shifts for the *CPT* and *N* resonances force the maximum of the optical response away from  $\delta = \delta_0$ , creating an overall asymmetric lineshape. The  $B$  term, linear in the two-photon detuning  $\delta - \delta_0$ , accommodates this effect.

We fit the skew Lorentzian (Eq. (2)) to both experimental data and numerical calculations using the model described above with atomic parameters for the  $^{87}\text{Rb}$  atom. We find that the lineshape asymmetry  $B/A$  decreases as overall laser intensity increases in both cases (Fig. 3). This reduction of asymmetry with increasing optical power is consistent with our model of the

different multi-photon (nonlinear) natures of the *CPT* and *N* resonances. As a three-photon process, the *N* resonance contrast scales faster with laser power than that of the two-photon *CPT* process (see Eq. (1)); thus lineshape asymmetry is less pronounced at large total power. Furthermore, the line asymmetry is greatly reduced over the entire range when the +2 sideband is suppressed (in our experiments by approximately 85% in intensity) *before* the light enters the atomic vapor cell. Our model indicates that reduction of the +2 sideband greatly inhibits the *CPT* resonance. The asymmetry  $B/A$  in the two photon line shape is proportional to the ratio of the rabi frequencies  $\Omega_2/\Omega_0$  and decreases with increasing ground state population difference. This connection between the parameterization of Eq. (2) and the full model explains the qualitative features of Figures 3, 4.

Another example, our model explains the difference between the observed lineshape asymmetries of *N+CPT* resonances on the  $D_1$  and  $D_2$  optical transitions [11]. Under identical conditions the *CPT* resonance has lower contrast on the  $D_2$  transition as compared to that of  $D_1$  [13]. This is a consequence of direct depolarization transitions ( $F = 2 \rightarrow F' = 3$ ) for the  $D_2$  drive and the resultant suppression of optical pumping of the ground state. Under identical conditions, the  $D_2$  *N* resonance is more symmetrical and has higher contrast than the  $D_1$  *N* resonance (Fig. 4).

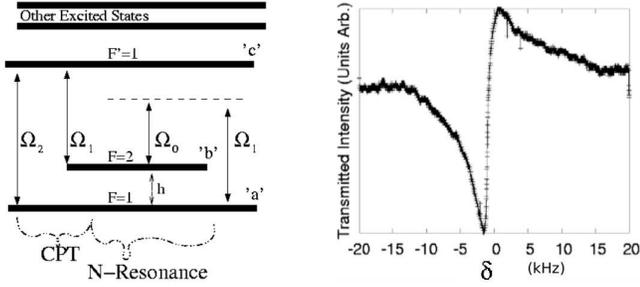
In conclusion, we have shown that a characteristic two-photon lineshape asymmetry arises in *CPT* and *N* resonances due to modulation sidebands in the interrogating laser light. A simple model for the combined effect of these optical fields in the joint *N+CPT* system explains quantitatively many observed features of the lineshape asymmetry. Temporal variation in lineshape asymmetry contributes to clock frequency instability. The effect described here is most relevant for *N* resonance-based clock stability, but can contribute also to the optical response of *CPT*-based clocks as well when driving fields are not perfectly balanced. More experimental studies are underway to fully assess the challenge of using a significantly distorted lineshape

in a clock. This asymmetry is generic to  $N$  resonance-based clocks using current-modulated laser diodes.

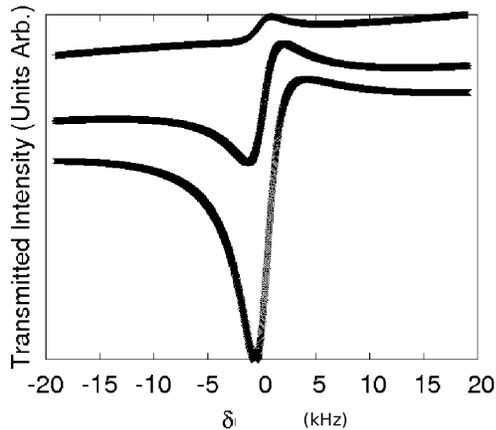
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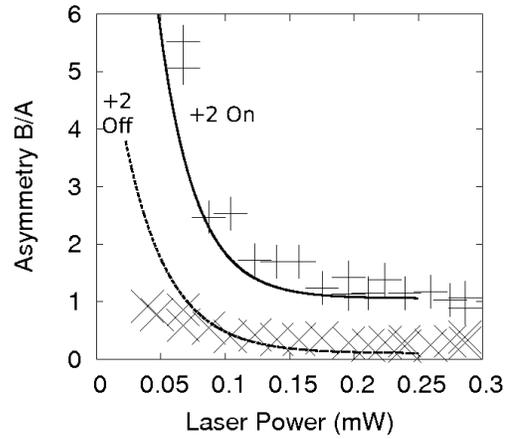


**Fig. 1:**  
 (a) Simplified level diagram with applied fields  $\Omega_0$  (carrier) and  $\Omega_1, \Omega_2$  (sidebands). (b) Example experimental  $N+CPT$  joint resonance, illustrating the typical asymmetry of the transmitted probe field  $\Omega_1$  lineshape in the presence of the  $\Omega_2$  sideband. The x-axis is the two-photon detuning. The laser power is .088 mW and the one photon detuning is about 350 MHz

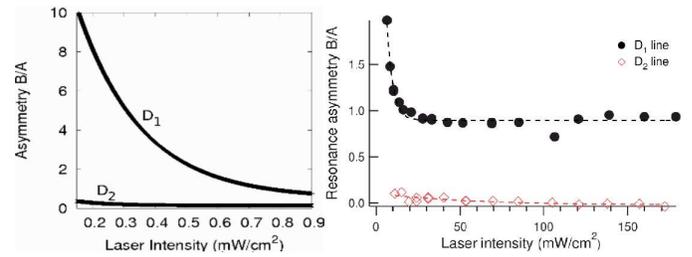


**Fig. 2:**  
 Numerical calculations of the probe field  $\Omega_1$  transmission intensity for  $CPT$  (top),  $N$  (bottom) and joint  $N+CPT$  resonances (middle), vertically offset for clarity. All parameters are for the  $^{87}\text{Rb}$  atom with  $\Omega_0 = 1$  MHz for the middle and bottom traces and  $\Omega_1 = 100$  kHz for all traces and  $\Omega_2 = 6$  kHz for the top and middle traces. The one-photon detuning used in these calculations is 100 MHz and so the background transmitted signal  $T(\Delta)$  is not level. The x-axis is two-photon detuning.

**Fig. 3:**



Lineshape asymmetry  $B/A$  for  $N+CPT$  resonances on the  $D_1$  transition of  $^{87}\text{Rb}$  with and largely without (85% reduced intensity) the +2 sideband. Numerical calculations based on our model are the lines, the “+” and “x” are the associated fitted  $B/A$  from experimental data.



**Fig. 4:**  
 Comparison of  $N+CPT$  lineshape asymmetry for  $D_1$  and  $D_2$  transitions: (a) Numerical calculations and (b) experiment (data from [11]). Difference with Figure 3 is due to the different experimental parameters (buffer gas pressure, laser intensities and detunings) of [11] that we have used in these calculations. The range of intensities for which there is experimental data is not directly accessible to our theory model, but both model and experiment indicate saturation of the two-photon optical pumping at high power.