Bound on Lorentz and CPT Violating Boost Effects for the Neutron


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A search for an annual variation of a daily sidereal modulation of the frequency difference between colocated $^{129}$Xe and $^3$He Zeeman masers sets a stringent limit on boost-dependent Lorentz and CPT violation involving the neutron, consistent with no effect at the level of 150 nHz. In the framework of the general standard-model extension, the present result provides the first clean test for the fermion sector of the symmetry of spacetime under boost transformations at a level of $10^{-27}$ GeV.

The invariance of the laws of nature under transformations between inertial reference frames that differ in relative velocity (“boosts”) is an essential feature of Lorentz symmetry, along with invariance under rotations. Experimental tests of boost invariance, such as the Kennedy-Thorndike experiment [1], have been performed for many years with increasing precision [2]. These experiments typically search for a variation of the velocity of light with the laboratory velocity and test boost invariance in the context of classical electrodynamics. However, the fundamental role of Lorentz symmetry in physics makes it desirable to test boost invariance for other systems, such as massive particles with spin.

In this Letter, we present a high-sensitivity experimental test of boost invariance for the neutron, which we interpret in the context of the standard-model extension (SME) [3], a general theoretical framework that allows a comprehensive and systematic study of the implications of Lorentz-symmetry violation at observable energies. The SME provides a widely accepted formalism for the interpretation and comparison of experimental searches for violations of Lorentz symmetry and associated violations of CPT symmetry (the product of charge conjugation, parity inversion, and time reversal). The SME has been applied to many systems, including mesons [4], photons [2,5], and leptons [6–8], as well as the neutron [9,10] and proton [11]. An observable Lorentz violation could be a remnant of Planck-scale physics. One attractive origin is spontaneous Lorentz breaking in a fundamental theory [12], but other sources are possible [13].

Our experiment consists of a long-term monitoring of the frequencies of colocated $^3$He and $^{129}$Xe Zeeman masers as the Earth rotates and revolves around the Sun. We search for a specific signature of a violation of boost invariance: an annual variation of the nuclear Zeeman splitting, modulated at the frequency of the Earth’s daily sidereal rotation. Such an effect could arise from couplings of the $^3$He and $^{129}$Xe nuclear spins (each largely determined by a valence neutron) to background tensor fields, including a dependence of the Zeeman frequencies on the instantaneous velocity (magnitude and direction) of the laboratory. The appeal of the noble-gas maser experiment is the excellent absolute frequency stability [10,14,15], and thus the sensitivity to small, slow variations in the magnitude of Lorentz-violating spin couplings.

Using the two-species noble-gas maser, we recently constrained the possible rotation-symmetry-violating couplings of the neutron spin with respect to an inertial reference frame based on the Earth [10]. Here, we choose a Sun-based inertial reference frame, which allows us to study cleanly—for the first time in the fermion sector—the symmetry of spacetime with respect to boost transformations. (The pioneering work of Berglund et al. [16] does not distinguish between the boost and rotation effects to which it is sensitive.) Our experiment’s rest frame moves with the Earth around the Sun at a velocity of magnitude $v_\oplus/c = 0.3 \times 10^{-5}$, and the Lorentz transformation that describes the change of coordinates from the laboratory frame to the Sun-based frame includes both a rotation, $\vec{R}$, and a boost along the velocity $\vec{\beta}$.

The most general, coordinate-independent Hamiltonian, $H$, containing the Zeeman effect (from an applied magnetic field $\vec{B}$) and Lorentz-symmetry-violating couplings of the noble-gas nuclear spins, $\vec{I}$, including leading terms to first order in $\vec{B}$, takes the simple form

$$H = \vec{I} \cdot (\gamma \vec{B} + \vec{R}(t)\vec{\Lambda}_0 + \vec{\beta}(t)\vec{R}(t)\vec{\Lambda}_0).$$  (1)

Here, the vectors $\vec{I}$ and $\vec{B}$ are expressed in the lab frame, whereas the explicit Lorentz-symmetry-violating vector $\vec{\Lambda}_0$ and $3 \times 3$ matrix $\vec{\Lambda}_0$ have elements that are combinations of SME coefficients, which may be determined in terms of fundamental Lorentz-violating interactions [17,18], and are assumed constant in the Sun frame. The second term of Eq. (1) leads to a rotation-dependent modulation of the maser frequency. The third term contains cross couplings, in which the rotation induces daily sidereal modulations of the maser frequencies, while the boost transformation induces a sinusoidal variation of the...
In this equation, \( J \) is the angular momentum. The Earth's sidereal angular rotation frequency is \( \Omega_\odot \), and \( \gamma \) is the diurnal systematic variation. We define \( \delta \) to be the deviation of the Earth's rotation from a circular orbit. For a sufficiently high flux of population-inverted noble-gas atoms, the Earth's rotation sensitivity is given by

\[
\delta = \beta_\odot (\sin \Omega_\odot T - \cos \gamma \cos \Omega_\odot T - \sin \gamma \cos \Omega_\odot T) \quad (3)
\]

Here, \( \Omega_\odot \) is the angular frequency of the Earth's orbital motion. The time \( T \) is measured by a clock at rest at the origin, with \( T = 0 \) taken at 2:35 AM (U.S. Eastern Standard Time), March 20, 2000 [23]. The angle between the Earth's equatorial plane and the Earth's orbital plane is \( \eta = 23.4^\circ \). We have ignored the laboratory's velocity due to the rotation of the Earth, whose magnitude is \( \beta_e = r_e \omega_\odot \sin \chi / c \approx 1.1 \times 10^{-6} \) (where \( r_e \) is the radius of the Earth), is 2 orders of magnitude smaller than the orbital velocity.

Because of the generally weak interactions of noble-gas atoms with the walls and during atomic collisions, the species can be maintained indefinitely.

For the boost-symmetry test, we choose a set of laboratory coordinates \((x, y, z)\), such that the \( x \) axis points south, the \( y \) axis points east, and the \( z \) axis points vertically upwards in the laboratory [22]. We assume that the Lorentz-violating coefficients of \( \lambda_\odot \) and \( \lambda_\odot \) are static and spatially uniform in the Sun frame, at least over the course of a solar year. The corresponding coefficients in the laboratory frame thus acquire a time dependence due to both the Earth's rotation and its revolution around the Sun. We also assume observer Lorentz covariance; hence direct Lorentz transformations yield the coefficients in the laboratory frame.

In the boost-symmetry test, we used the noble-gas maser as a comagnetometer to stabilize the magnetic field, which was oriented along the \( y \) axis (i.e., west to east). Thus the leading Lorentz-violating frequency variation of the free-running noble-gas maser was given by

\[
\delta \nu_{Xe} = \delta \nu_\lambda \sin \omega_\odot T_\odot + \delta \nu_\gamma \cos \omega_\odot T_\odot \quad (4)
\]

where

\[
\delta \nu_\lambda = k [\lambda_\odot + \beta_\odot (\lambda_{ss} \sin \Omega_\odot T + \lambda_{cc} \cos \Omega_\odot T)] \quad (5)
\]

\[
\delta \nu_\gamma = k [\lambda_\odot + \beta_\odot (\lambda_{ss} \sin \Omega_\odot T + \lambda_{cc} \cos \Omega_\odot T)]
\]

Here \( \lambda_{ss}, \lambda_{cc}, \lambda_{sc}, \ldots \) are combinations of Sun-frame Lorentz-violating coefficients of \( \lambda_\odot \) and \( \lambda_\odot \), and \( k = -8.46 \times 10^{32} \) nHz/GeV [10].

We note that Eqs. (4) and (5) cleanly distinguish the effects of rotation alone (terms proportional to \( \lambda_\odot \) and \( \lambda_\odot \)) from the effects of boosts due to the Earth's motion (terms proportional to \( \lambda_{ss}, \lambda_{cc}, \lambda_{sc}, \lambda_{cc} \)). In addition, these equations indicate that the sensitivity of our experiment to violations of boost-symmetry is reduced by a factor of \( \beta_e \approx 10^{-4} \) with respect to the sensitivity to rotation-symmetry violation. However, for models of Lorentz violation that are isotropic in the frame of the cosmic microwave background [24], our experiment has greater sensitivity to boost-symmetry violation than to rotation-symmetry violation.

As discussed in [10], we acquired noble-gas maser data in four different runs spread over about 13 months (see Fig. 1). Each run lasted about 20 d, and we reversed the direction of the magnetic field after the first \( \sim 10 \) d in each run to help distinguish possible Lorentz-violating effects from diurnal systematic variations. We fit this data to Eq. (4). Table I lists, for each run, the mean values we

\[
R^{ij} = \begin{pmatrix}
\cos \chi \cos \omega_\odot T_\odot & \cos \chi \sin \omega_\odot T_\odot & -\sin \chi \\
-\sin \omega_\odot T_\odot & \cos \omega_\odot T_\odot & 0 \\
\sin \chi \cos \omega_\odot T_\odot & \sin \chi \sin \omega_\odot T_\odot & \cos \chi
\end{pmatrix} \quad (2)
\]
the system in which there is no daily sidereal modulation of the $^3$He-maser frequency due to Lorentz-violating coefficients in the $\hat{X}$ and $\hat{Y}$ directions (Sun-centered frame). For each run, $\delta \nu_X$ and $\delta \nu_Y$ correspond to a very good approximation to a single high-precision measurement of the $X$ and $Y$ components of $\delta \nu_{3\text{He}}$ performed at the mean time $T_j$.

Next, we fit the values of $\delta \nu_X$, $\delta \nu_Y$, and $T_j$ in Table I to Eq. (5), thus obtaining the fit parameters reported in Table II and shown graphically in Fig. 1. We treated all fit parameters as independent and we extracted energy bounds for Lorentz-violating coefficients disregarding the possibility of accidental mutual cancellations. This analysis yielded no significant violation of boost invariance, with a limit of about 150 nHz on the magnitude of an annual modulation of the daily sidereal variation in the $^3$He-maser frequency.

To confirm that our result is consistent with the null hypothesis (i.e., no Lorentz-violating effect), we performed two checks. First, we generated 10 000 faux $^3$He-maser data sets including sidereal day frequency variations drawn from a normal distribution of zero mean but with standard errors for $\delta \nu_X$ and $\delta \nu_Y$ at each time $T_j$ equal to the corresponding values found in the experiment. For each faux data set, we calculated the $\chi^2$ of the fit to Eq. (5) and found that the value $\chi^2 = 0.30$ from the real experimental data is highly probable for a system in which there is no daily sidereal modulation of the $^3$He-maser frequency at the experiment’s level of sensitivity. In the second check, we performed a series of F tests to estimate the probabilities that the values of the fit parameters, determined from the maser data, arise entirely from statistical fluctuations. For all fit parameters, the F tests yielded probabilities greater than 30%, whereas it is customary to consider that a fit parameter is significantly different from zero only if the F test probability is smaller than 5% or 1%.

We also performed a series of checks for systematic effects, including sidereal day and year variations in maser temperature and signal amplitude (e.g., driven by variations in the optical-pumping laser). Temperature fluctuations in the $^3$He and the $^{129}$Xe detection circuit can induce small maser-frequency shifts. Accurate temperature monitoring over the course of the 13-month experiment showed a maximum 1.6 mK sidereal day variation of maser temperature, corresponding to a maximum sidereal day $^3$He-maser-frequency modulation of about 4 nHz, which is an order of magnitude smaller than our statistical sensitivity. A careful analysis of the maser amplitude showed a lack of phase coherence in sidereal day modulations over the 13-month data set, and hence an insignificant systematic sidereal-year variation in the $^3$He-maser frequency.

To interpret this test of boost invariance, we follow the conventions of Ref. [25], Appendix C, which allows us to relate the maser frequencies to the various SME coefficients for Lorentz and CPT violations. In particular, the neutron—and hence the frequency of each noble-gas maser—is sensitive to Lorentz and CPT violations controlled by the SME coefficients $b_\Lambda$, $d_{\Lambda \Sigma}$, $H_{\Lambda \Sigma}$, and $g_{\Lambda \Sigma \Gamma}$ [17]. Table II shows the corresponding bounds provided by our experiment to combinations of Sun-frame SME coefficients, including the clean limit of $\sim 10^{-27}$ GeV on boost violation.

In conclusion, we used colocated $^3$He and $^{129}$Xe Zeeman masers to perform a high-sensitivity search for a violation of boost invariance of the neutron. We found no significant sidereal annual variation in the free-running $^3$He-maser frequency at a level of approximately 150 nHz. This result provides the first clean test of boost symmetry for a fermion, and, in the context of the general standard-model extension, places a bound of about $10^{-27}$ GeV on 11 previously unexplored coefficients among the 44 coefficients describing possible leading-order Lorentz- and CPT-violating couplings of the neutron. Significant improvements may be possible with a

![FIG. 1. Time course of the mean values of $\delta \nu_X$ and $\delta \nu_Y$. For each plot the dashed line is the best fit obtained from Eq. (5), using the fit parameters $\lambda_\Lambda$, $\Lambda_\Lambda$, $\Lambda_{\Sigma \Sigma}$, $\Lambda_{\Sigma \Gamma}$, and $\Lambda_{\Gamma \Gamma}$ (see also the fit results in Table II). Dotted lines indicate the 1σ confidence bands for the fit model.](image-url)
TABLE II. Limits from the present work on Lorentz violation of the neutron, expressed in terms of (i) the fit parameters of Eqs. (4) and (5), i.e., coefficients for the general Lorentz-symmetry-violating vector $\lambda_6$ and $\Lambda_6$ (both in the Sun frame); and (ii) combinations of Sun-frame SME coefficients for Lorentz and CPT violation (defined in Appendix B of Ref. [18]). Bounds on rotation-symmetry violation are set by the limits on $\lambda_c$ and $\lambda_s$, whereas bounds on boost-symmetry violation are determined from $\Lambda_{XX}$, $\Lambda_{YY}$, $\Lambda_{ZZ}$, and $\Lambda_{XY}$.

<table>
<thead>
<tr>
<th>Measurable combinations of SME coefficients</th>
<th>Fit parameters</th>
<th>Fit results (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{b}_Y - 0.0034\tilde{d}<em>Y + 0.0034\tilde{g}</em>{DY}$</td>
<td>$\lambda_c$</td>
<td>$(8.0 \pm 9.5) \times 10^{-32}$</td>
</tr>
<tr>
<td>$-\tilde{b}_Y + 0.0034\tilde{d}<em>Y - 0.0034\tilde{g}</em>{DX}$</td>
<td>$\lambda_s$</td>
<td>$(2.2 \pm 7.9) \times 10^{-32}$</td>
</tr>
<tr>
<td>$-\cos\theta[\tilde{b}<em>Y + \frac{1}{2}\tilde{d}<em>Y - \tilde{g}</em>{XY} + (\tilde{g}</em>{YY} - 2\tilde{d}<em>Y + \frac{1}{2}\tilde{d}<em>Q)] + \sin\theta(\tilde{d}</em>{XZ} - \tilde{H}</em>{XZ})$</td>
<td>$\Lambda_{XZ}$</td>
<td>$(-1.1 \pm 1.0) \times 10^{-27}$</td>
</tr>
<tr>
<td>$-\tilde{H}_{YZ}$</td>
<td>$\Lambda_{YX}$</td>
<td>$(0.2 \pm 1.8) \times 10^{-27}$</td>
</tr>
<tr>
<td>$[\frac{1}{2}\tilde{b}<em>Y + \frac{1}{2}\tilde{d}<em>Y - \tilde{g}</em>{XY} - \frac{1}{2}\tilde{g}</em>{XT}](\tilde{g}_{YT} - 2\tilde{d}_Y + \frac{1}{2}\tilde{d}_Q]$</td>
<td>$\Lambda_{XY}$</td>
<td>$(-1.8 \pm 1.9) \times 10^{-27}$</td>
</tr>
<tr>
<td>$\cos \theta(\tilde{H}<em>{ZT} - \tilde{d}</em>{ZT}) - \sin \theta \tilde{H}_{YT}$</td>
<td>$\Lambda_{Xc}$</td>
<td>$(-1.1 \pm 0.8) \times 10^{-27}$</td>
</tr>
</tbody>
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21Ne/3He Zeeman maser [26], with masers located on a rotating table [27], or with space-based clocks [28].

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